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SYNTHESIS OF DUO-CHANNEL EVEN-ORDER PHASE FILTERS

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The method of synthesis of odd-order duo-channel phase filters, which uses the theory of quadripoles, is described in the paper [1]. However, the base principle of such method does not allow its easy use for even-order duo-channel phase filters synthesis, unfortunately.

Using the theory of quadripoles, the fundamental opportunity of synthesis of even-order phase filters is described in this article. The frequency and performance characteristics are provided. Additionally, the special cases of significant simplification of realizations during modelling of such phase filtration system in real time are given. Finally, the possibility of design of even-order duo-channel phase filters on bilines is shown.

Keywords: *phase filter, synthesis, digital filters*

1. Introduction

The method of synthesis of odd-order duo-channel phase filters, which uses the theory of quadripoles, is described in this paper [1]. However, unfortunately, the base principle of such a method does not allow its easy use for similar even-order duo-channel phase filters synthesis.

Using the theory of quadripoles, the fundamental opportunity of synthesis of even-order phase filters is described in this article. The frequency characteristics are also provided.

2. Description of Method

It is known that the physically realizable analogue polynomial prototypes of even-order Butterworth or Chebyshev type I filters with equal input and output loads belong to electrical-asymmetric circuits. The method of synthesis of duo-channel even-order phase circuits, which is suggested below, is based on the artificial leading of a structural- and electrical-asymmetric circuit (Fig. 1) to a structural- and electrical-symmetric circuit. The subsequent known transformation of transfer function of a modified circuit allows it to be realized as a duo-channel system on phase units [1].

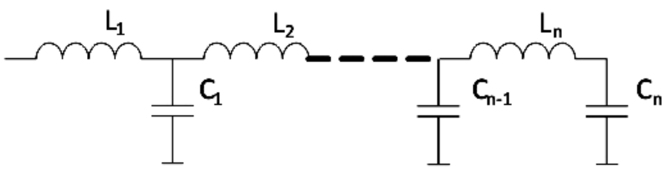


Figure 1. LC-prototype of electrical- and structural-asymmetric circuit

The simplest reversible transformation of an electrical-asymmetric circuit into an electrical-symmetric circuit is based on a multiplication of A-matrix of the circuit by the complex J-matrix:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} = j \begin{bmatrix} a_{12} & a_{11} \\ a_{22} & a_{21} \end{bmatrix} \qquad (1)$$

here the a_{12} is equal to a_{21}.

Following (1), one of the possible variants of a LC-prototype realization of electrical-symmetric even-order circuit is the following prototype structure (Fig. 2):

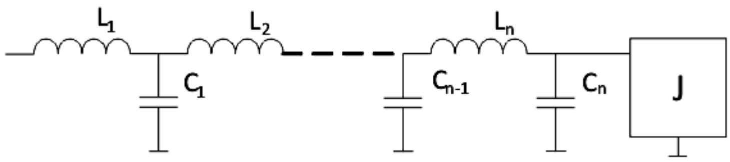


Figure 2. LC-prototype of an electrical-symmetric circuit

Since only the structural-symmetric circuit might be realized in the form of duo-channel phase system, it is necessary to modify the structure shown on Fig. 2, as follows (Fig. 3):

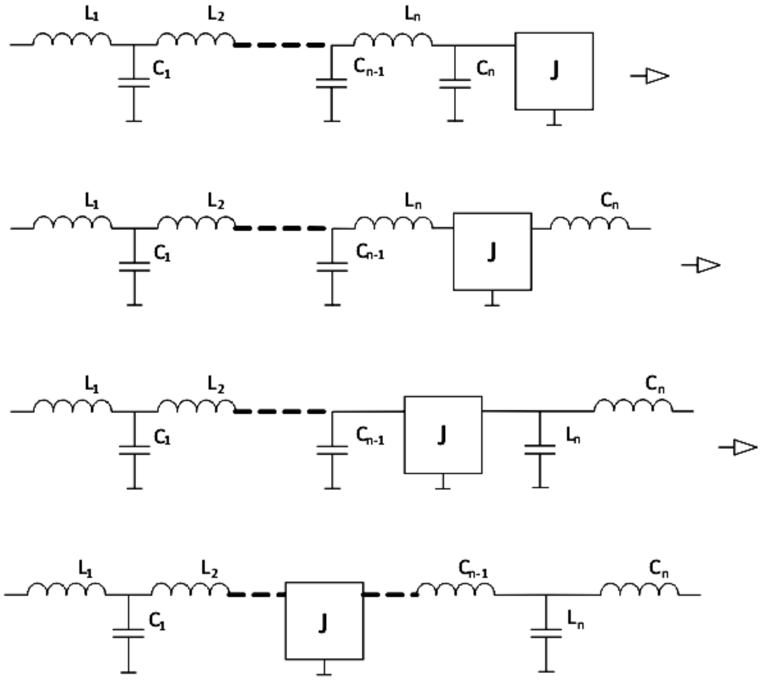


Figure 3. Transformation of the electrical-symmetric structural-asymmetric circuit into structural-symmetric

It can be shown that the normalized factors of such a circuit (Fig. 3) are pair-equal for Butterworth or Chebyshev Type I filters, i.e. $L_1 = C_n$, $C_1 = \underline{L_n}$, $L_2 = C_{n-1}$ etc [2]. Besides it is known that the complex J-matrix might be realized by one of two ways (Fig. 4):

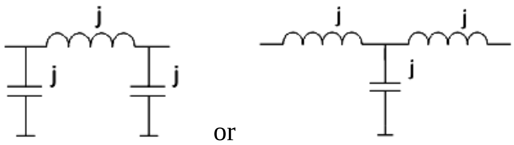


Figure 4. Variants of J-matrix realization

For simplicity we will consider the electrical- and structural-asymmetric circuit of 4th-order (Fig. 5):

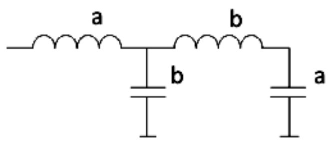


Figure 5. LC-prototype of an asymmetric 4th-order circuit

Here the *a* and *b* are the normalized circuit’s parameters. Following Fig. 4 and Fig. 5, the modified structural-symmetric 4th-order circuit might be represented, for example, in the following way (Fig. 6):

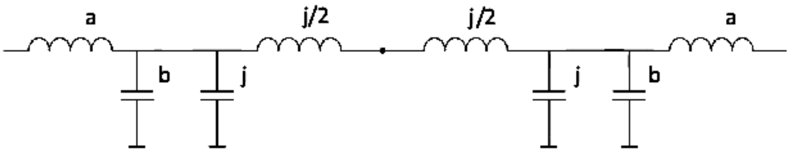


Figure 6. Structural-symmetric circuit with insertion of j-elements

Half of the circuit shown on Fig. 6, can be described through factors of its A-matrix:

$$[A_{half}]=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}=\begin{bmatrix}1 & pa\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ pb & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ j & 1\end{bmatrix}\begin{bmatrix}1 & j/2\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1+p^2ab & pa\\ pb & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ j & 1\end{bmatrix}\begin{bmatrix}1 & j/2\\ 0 & 1\end{bmatrix}=$$

$$=\begin{bmatrix}1+p^2ab+jpa & pa\\ pb+j & 1\end{bmatrix}\begin{bmatrix}1 & j/2\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1+p^2ab+jpa & \frac{1}{2}(j+jp^2ab+pa)\\ pb+j & \frac{1}{2}(jpb+1)\end{bmatrix}. \quad (2)$$

It can be shown [1], that the transfer function of any structural-symmetric circuit with equal input and output loads can be represented as a sum of transfer functions of two phase channels. Hence:

$$H=\frac{1}{2}\left(\frac{a_{11}-a_{21}}{a_{11}+a_{21}}+\frac{a_{22}-a_{12}}{a_{22}+a_{12}}\right)=$$

$$=\frac{1}{2}\left(\frac{1+p^2ab+jpa-pb-j}{1+p^2ab+jpa+pb+j}+\frac{jpb+1-j-jp^2ab-pa}{jpb+1+j+jp^2ab+pa}\right)=$$

$$=\frac{1}{2}\left(\frac{p^2-p\left(\frac{b-ja}{ab}\right)+\left(\frac{1-j}{ab}\right)}{p^2+p\left(\frac{b+ja}{ab}\right)+\left(\frac{1+j}{ab}\right)}-\frac{p^2-p\left(\frac{b+ja}{ab}\right)+\left(\frac{1+j}{ab}\right)}{p^2+p\left(\frac{b-ja}{ab}\right)+\left(\frac{1-j}{ab}\right)}\right). \quad (3)$$

Based on the previous computations, a general formula for the transfer function of the even-order duo-channel phase filters can be derived. As has been shown before, the suggested approach is based on artificial leading of a structural-asymmetric circuit to a structural-symmetric circuit. Since it is sufficient to use the only half of the entire even-order circuit to evaluate its transfer function, the A-matrix of the half of initial LC-circuit in conjunction with the half of J-matrix can describe this half. Therefore the half of the entire circuit is described by the following A_{half}-matrix (5):

$$[A_{half}]=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}=[A]\cdot\left[J/2\right]=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\cdot\begin{bmatrix}1 & j/2\\ j & 1/2\end{bmatrix}=\begin{bmatrix}A_{11}+jA_{12} & \frac{1}{2}(jA_{11}+A_{12})\\ A_{21}+jA_{22} & \frac{1}{2}(jA_{21}+A_{22})\end{bmatrix}. \quad (4)$$

Following [1] and (4), the entire transfer function of the structural-symmetric even-order phase circuit might be described in the following way:

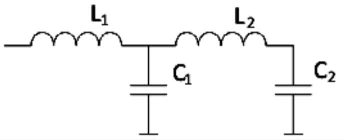
$$H=\frac{1}{2}\left(\frac{a_{11}-a_{21}}{a_{11}+a_{21}}+\frac{a_{22}-a_{12}}{a_{22}+a_{12}}\right)=$$

$$=\frac{(A_{11}-A_{21})+j(A_{12}-A_{22})}{(A_{11}+A_{21})+j(A_{12}+A_{22})}-\frac{(A_{11}-A_{21})-j(A_{12}-A_{22})}{(A_{11}+A_{21})-j(A_{12}+A_{22})}=$$

$$=2j\frac{\sqrt{(A_{11}-A_{21})^2+(A_{12}-A_{22})^2}}{\sqrt{(A_{11}+A_{21})^2+(A_{12}+A_{22})^2}}\sin\left(arctg\left(\frac{A_{12}-A_{22}}{A_{11}-A_{21}}\right)-arctg\left(\frac{A_{12}+A_{22}}{A_{11}+A_{21}}\right)\right). \quad (5)$$

3. Examples

As an example, let us consider the analogue polynomial prototype of Chebyshev type I 4^{th}-order filter. Below the normalized values of filter elements at equal input and output loads are given (a_{max} = 0.28 dB) [2]:



L_1 (a)	1.146
C_1 (b)	1.513
L_2 (b)	1.513
C_2 (a)	1.146

The module of amplitude-frequency response of LC-prototype of the aforementioned structural-asymmetric circuit is show in Fig. 7.

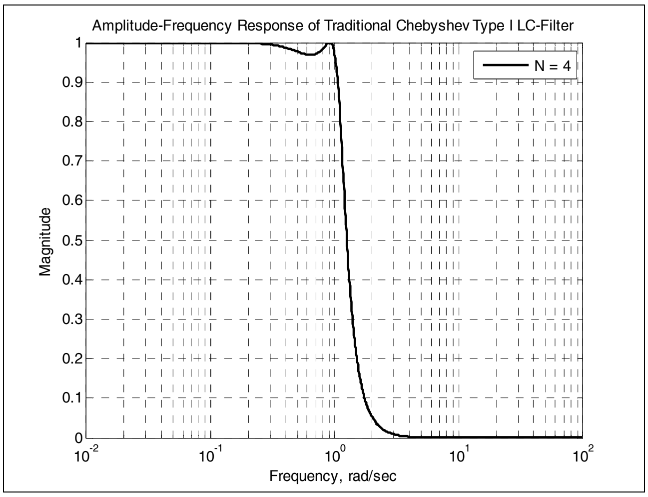


Figure 7. The module of amplitude-frequency response of Chebyshev type I 4^{th}-order filter

Accordingly, the module of amplitude-frequency response of a duo-channel 4^{th}-order phase system is shown on Fig. 8.

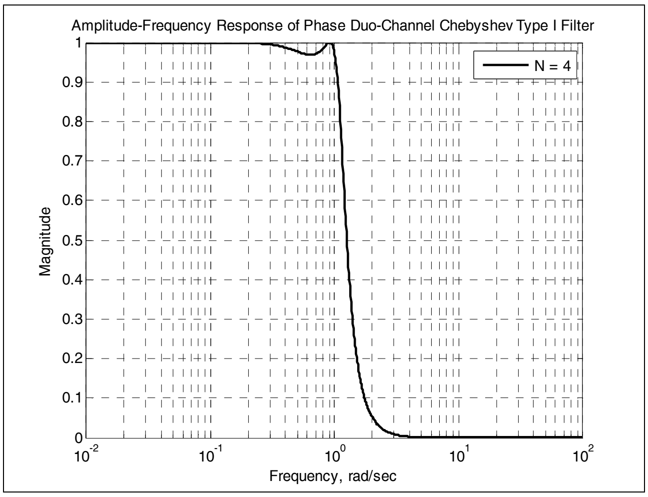
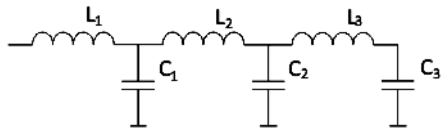


Figure 8. The module of amplitude-frequency response of duo-channel Chebyshev type I 4^{th}-order filter on phase units

It can be also shown that these modules of amplitude-frequency responses (Fig. 7 and 8) are accurate within machine arithmetic error.

As a second example, let us show the normalized factors of the 6th-order Chebyshev Type I filter, as well as its traditional amplitude-frequency response (simple LC-model, Fig. 9) and the amplitude-frequency response of the duo-channel phase implementation (Fig. 10):



L_1 (a)	1.036
C_1 (b)	1.516
L_2 (c)	1.878
C_2 (c)	1.878
L_3 (b)	1.516
C_3 (a)	1.036

In this case the final transfer function of the duo-channel phase system is the following (6):

$$H = \frac{1}{2}\left(\frac{a_{11} - a_{21}}{a_{11} + a_{21}} + \frac{a_{22} - a_{12}}{a_{22} + a_{12}}\right) =$$

$$\frac{1}{2}\left(\frac{p^2ab - pb + 1 + j(p^3abc - p^2bc + p(a+c) - 1)}{p^2ab + pb + 1 + j(p^3abc + p^2bc + p(a+c) + 1)} - \frac{p^3abc - p^2bc + p(c+a) - 1 - j(pb - 1 - p^2ab)}{p^3abc + p^2bc + p(c+a) + 1 + j(pb + 1 + p^2ab)}\right). \quad (6)$$

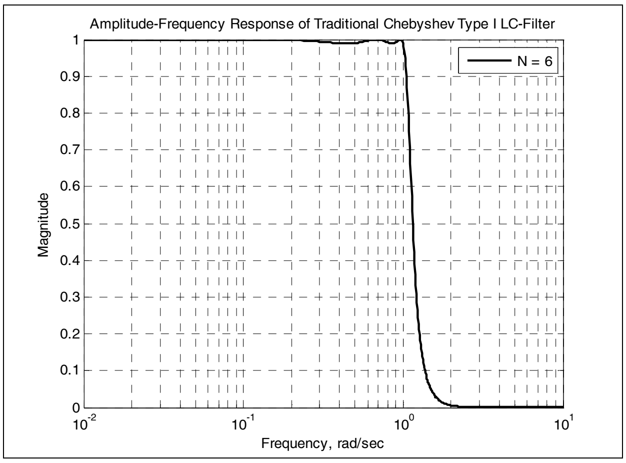


Figure 9. The module of amplitude-frequency response of Chebyshev type I 6th-order filter

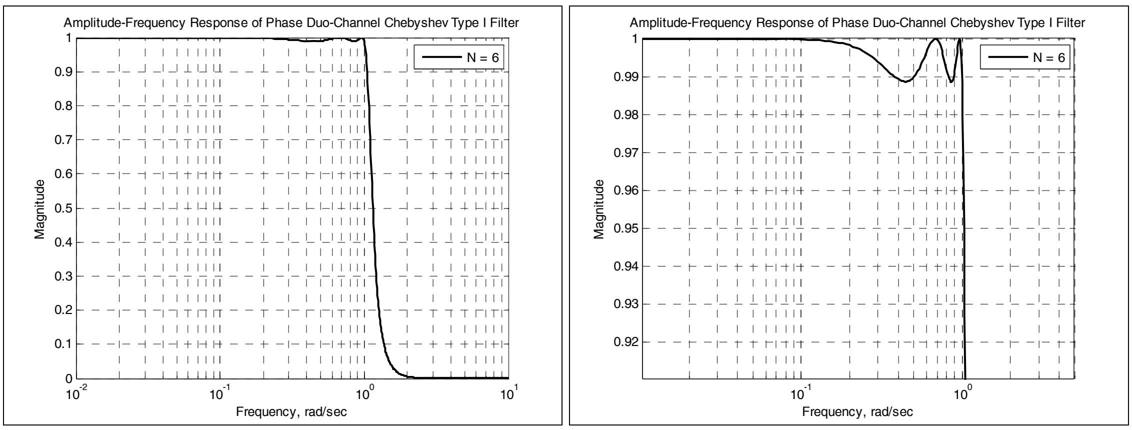


Figure 10. The module of amplitude-frequency response of duo-channel Chebyshev type I 6th-order filter on phase units

4. Conclusions

The fundamental opportunity of synthesis of duo-channel even-order phase filters, using the traditional matrix transformations of transfer function, is shown. Some examples are also provided. The frequency responses of the new structures are equal to the original within the whole frequency range.

References

1. Mamirov, T. Synthesis of High-Selective Phase Filters: master paper, Transport and Telecommunication Institute, Riga, 2003.
2. Hanzel, Grant E. *Filter Design and Evaluation*, Van Nostrand Reinhold Company, New-York, 1969.
3. Moshits, G., Horn, P. *Active Filters Design*. Moscow: Mir, 318 p, 1984. (In Russian)
4. Yeremeyev, V., Mamirov, T. Synthesis of Computational-Effective Structures of Digital Filters, *Transport and Telecommunication*, ISSN 1407-5806, 145 p.
5. Yeremeyev, V., Mamirov, T, Gumenyuk, A. New Structures of Undersensitive Digital Filters, *Computer Modelling and New Technologies*, ISSN 1407-5806, 145 p, 2001. (In Russian)