IDENTIFICATION OF NARROW-BAND DISCRETE SYSTEMS

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1. INTRODUCTION

Modern digital signal processing makes very strict demands to the filtration systems. Ones of them are high selectivity and narrowness of band.

Traditional bi-quad realizations [3] are not enough robust and also provide relatively low narrowness of band. Practically realizable normalized pass bands of a digital low pass bi-quad filter are only about 10^{-4}-10^{-7}($\tilde{\Omega}$). However, some practical applications, for example, in systems of artificial sense of smell, demand higher resolution of frequency components. Earlier by authors the new bi-lines digital structures have been offered, possessing super-narrowness of band - till 10^{-15}. The detailed analysis of such structures is given in the literature [2, 3].

Identification of bi-line structures in real time is extremely problematic. The traditional way of an estimation of dynamic frequency characteristics by the impulse response in this case is inconvenient because of its excessively big required length. It does not allow investigating frequency characteristics with necessary accuracy with the help of Fourier discrete transformation. The last causes practically unpredictable errors because of insufficient precision of machine arithmetic and excessively big number of required arithmetic operations.

2. DESCRIPTION OF THE RESEARCH

In this work the new technique of recursive systems identification is offered. It is based on synthesis of special finite input test influences, at which reaction of correctly designed filter has strictly limited duration.

As a base, we shall consider a transfer function of the digital filter in Z-area:

$$H(z^{-1}) = H_0 \frac{a_0 + a_1 z^{-1} + \ldots + a_N z^{-N}}{1 + b_1 z^{-1} + \ldots + b_N z^{-N}} \quad (1)$$

and formulate the next theorem.

Theorem. *For identification of any discrete N-order IIR-system it is always possible to generate an input influence with duration no more, than (N+1) samples, at which the finite length of output sequence is guaranteed.*

The proof. Really, the output response of any IIR-system of the finite order is:

$$Y(z^{-1}) = H(z^{-1}) \cdot X(z^{-1}).$$

If this product is a polynomial of z^{-1}, it is enough, that the reaction of such system will be finite length. It is possible only in a case, when $X(z^{-1})$ is equal to a polynomial of a transfer function denominator. At such input influence the Z-image of IIR-system reaction will be equal to numerator of transfer function, which is finite by definition.

For identification of the unknown system, having, in general, transfer function (1), generating of input unique sequence consists of samples selection of input influence, until output response becomes finite. After determination of both sequences a system's transfer function automatically becomes known. Then it is possible to calculate system frequency characteristics and to draw a conclusion about theirs compliance with goal requirements.

The offered method is especially useful for super narrow-band systems identification.

We shall consider the bi-line structure, which transfer function is described by expression:

$$H(p)=\prod_{i=1}^{N/2}\frac{p^2+A_i}{p^2+B_ip+C_i}=\prod_{i=1}^{N/2}\frac{(p-z_i)(p-\overset{*}{z}_i)}{(p-p_i)(p-\overset{*}{p}_i)}=\prod_{i=1}^{N/2}\left(h_{o1i}h_{o2i}\frac{(k-z_i)-(k+z_i)z^{-1}}{1-h_{o1i}(k+p_i)z^{-1}}\cdot\frac{(k-\overset{*}{z}_i)-(k+\overset{*}{z}_i)z^{-1}}{1-h_{o2i}(k+\overset{*}{p}_i)z^{-1}}\right)=$$

$$=\prod_{i=1}^{N/2}\left(h_o\frac{b_{0i}-b_{1i}z^{-1}}{1-\alpha_iz^{-1}}\cdot\frac{\overset{*}{b}_{0i}-\overset{*}{b}_{1i}z^{-1}}{1-\overset{*}{\alpha}_iz^{-1}}\right). \qquad (2)$$

For identification of this bi-line system it is expedient to form an input influence, which Z-transformation is equal to polynomials' product of denominators. In this case, while forming of input sequence the system's poles in Z-area (α_i) are selected.

3. EXAMPLES

Let's make identification[1] of elliptical bi-line filter, order of which is known (N=14). The theoretical impulse characteristic of such system is shown on fig. 1.

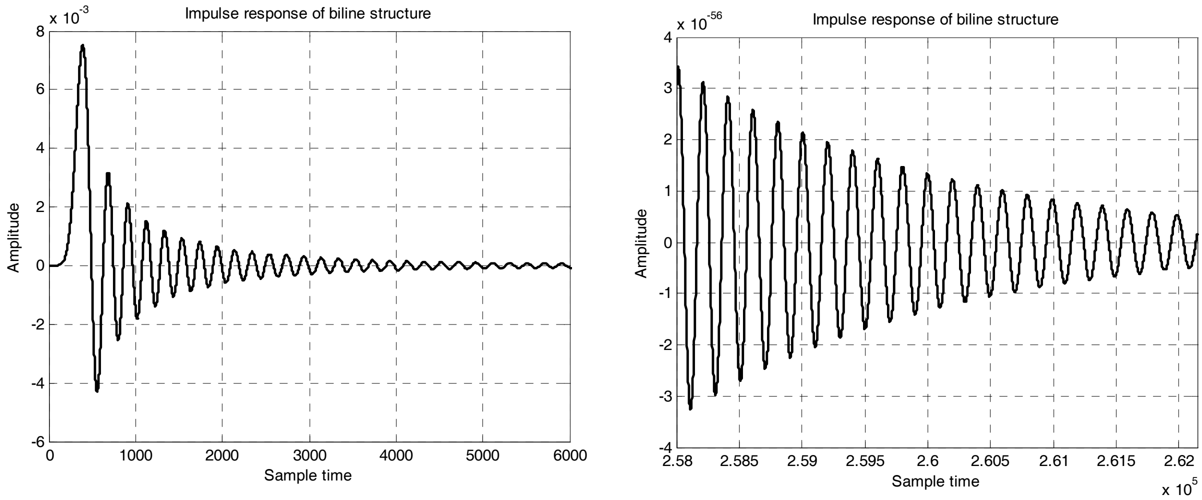

Fig. 1. *Impulse response of bi-line elliptical filter (N = 14)*

Unique input influence $X_{in}=\prod_{k=1}^{N/2}(1-\alpha_iz^{-1})(1-\overset{*}{\alpha}_iz^{-1})$ of the minimal length we shall determine either by random search of all poles α_i, or, that is certainly better, using known evaluated initial approach - with the help of methods of local optimisation.

As a result of the described identification process, the input influence of (N+1) samples length is determined (for presentation samples are resulted with the limited precision) (fig. 2):

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Xin = 1*10^3*[0.0010 -0.0140 0.0910 -0.3640 1.0010 -2.0020 3.0029 -3.4319
              3.0029 -2.0019 1.0010 -0.3640 0.0910 -0.0140 0.0010]
```

[1] Modelling of all processes was carried out on Pentium 4 2.4 GHz, 1 Gb RAM under OS Windows XP Professional, environment - Matlab 6.5 Release 13.

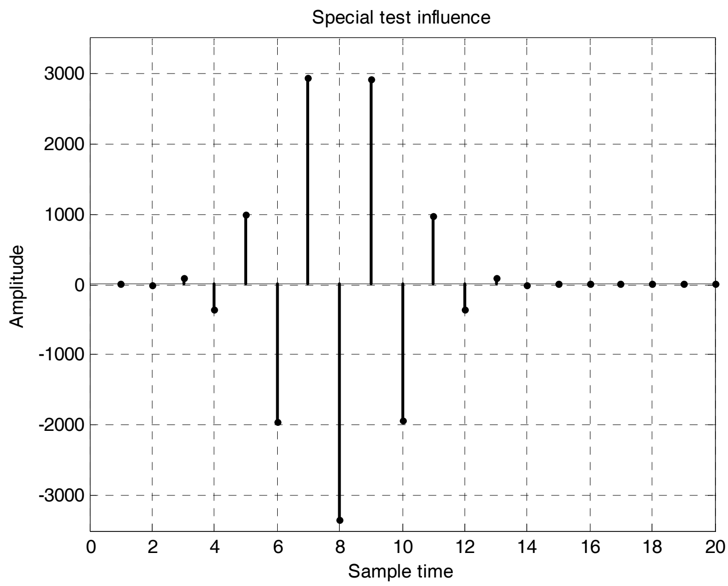

Fig. 2. *Formed input influence*

The system response is certainly finite (fig. 3):

Y_{out} = [0.0010 -0.0141 0.0904 -0.3586 0.9798 -1.9506 2.9179 -3.3317
2.9179 -1.9506 0.9798 -0.3586 0.0904 -0.0140 0.0010]

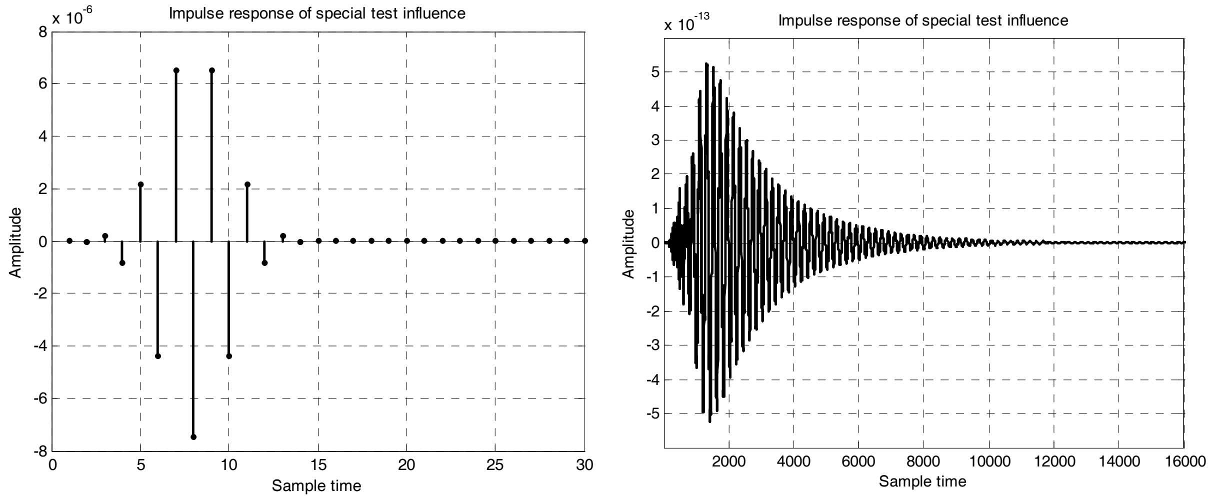

Fig. 3*. Response of elliptical filter on special input influence*

During system's identification in real time it's important not only to receive amplitude-frequency response (AFR), meeting the set requirements. In particular, sometimes it is necessary to analyse, in what degree allowable (by criterion of stability) random deviations of transfer function's poles influence on AFR's form and on target reaction. For example, we should add uniform noise N_0 to poles, so that in result they remain inside unit circle on Z – plane:

$$p_i = \alpha_i \cdot (1 + N_0') + j\beta_i (1 + N_0''),\ |p_i| \le 1 \cdot$$

On fig. 4, 5 the example of the selective system's identification with the added random additive polar noise about 10^{-2} % is shown. Apparently, output response, as expected, is finite; however "identified" AFR in a pass band is significantly distorted.

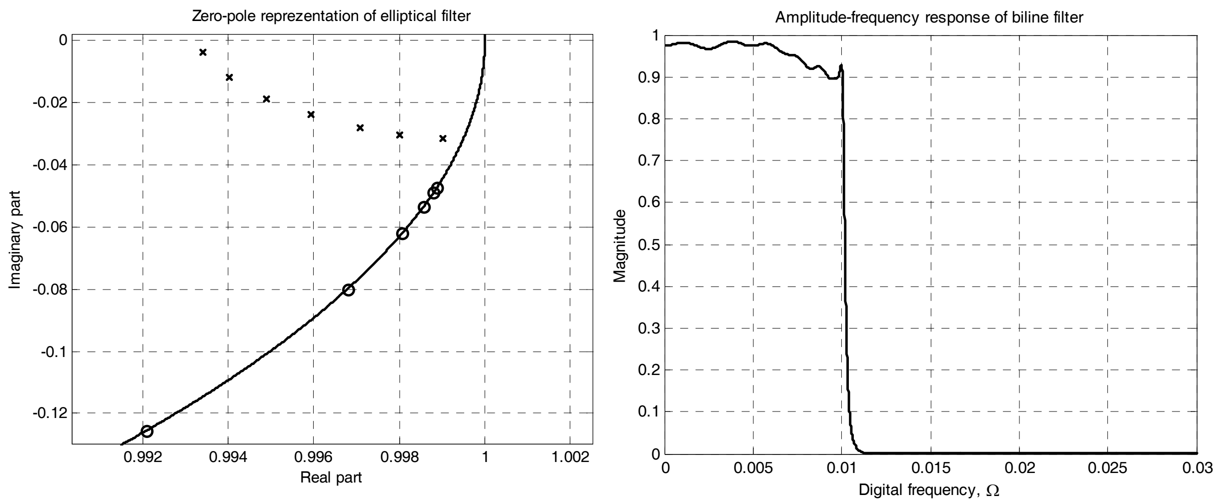

Fig. 4. *Poles and zeros arrangement of elliptical bi-line filter and it's AFR*

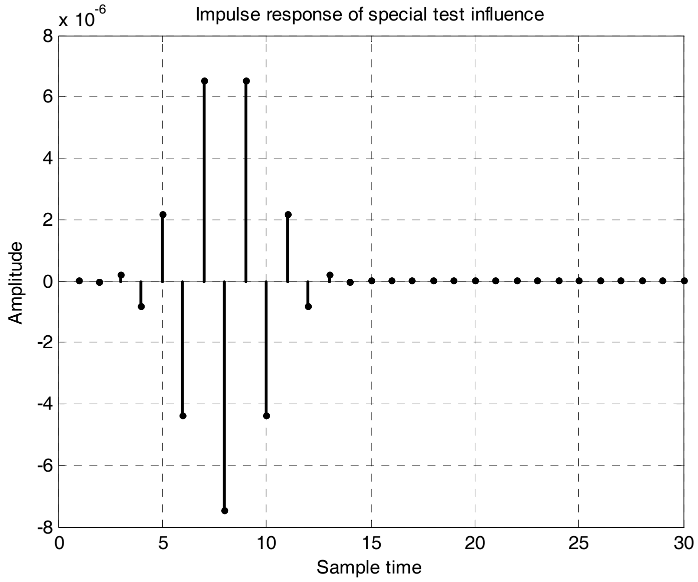

Fig. 5. *Test response of "noised" filter*

Let's consider a possible defragmentation of identification process in case of parallel program realization, with an opportunity of reception of separate sub-channels responses. Transfer function can be presented so:

$$H(z^{-1}) = H_0 \prod_{k=1}^{N/2} \frac{b_{0k} + b_{1k}\, z^{-1} + b_{2k}\, z^{-2}}{1 + a_{1k}\, z^{-1} + a_{2k}\, z^{-2}} = H_0^{'} \prod_{k=1}^{N/2} \frac{b_{0k} + b_{1k}\, z^{-1} + b_{2k}\, z^{-2}}{(1 - z_k z^{-1})(1 - \overset{*}{z}_k\, z^{-1})} = H_0^{''} \left(\sum_{k=1}^{N/2} \left(\frac{K_k z_k}{1 - z_k z^{-1}} + \frac{\overset{*}{K}_k\, \overset{*}{z}_k}{1 - \overset{*}{z}_k\, z^{-1}} \right) + 1 \right). \quad (3)$$

Apparently from expression (4), each part of the sum can be identified separately. In that case the problem of identification is paralysing and is technologically carried out faster in real time. It is enough to send the minimally short test complex sequence (two inputs - real and imaginary) on each sub-channel. Summarizing responses, it is possible to start an estimation of all transfer function.

4. CONCLUSIONS

The new method of identification of recursive, in particular, narrow-band systems is suggested. The examples of system identification with parameters are shown.

References

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