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Intelligent Transport Systems (Electronics)

RESEARCH OF THE HIGH-PERFOMANCE FERRITE DIRECTING SYSTEM

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The author of the article is going to make a research in order to obtain an analytical expression simple enough that allows producing the calculations of the electrical parameters of the ferrite directing system and its effectiveness on the phase of the projecting sketch.

Keywords: ferrite directing system, surface wave, electrical parameters

1. INTRODUCTION

The effectiveness of transforming the signal from generator to the load in the great extent is defined by the directing system.

The directing system's work is dependent on the rightfulness of the choice of its parameters. Besides, by working out the system one of the most important requirements are as follows: its small size.

Due to it is proposed in the article a compact cylindrical ferrite directing system without spiral. The important worth of the above-mentioned directing system is that its cuttings can be used as antennae.

Though, the precise mathematical analysis of the mentioned system turns out to be a very complicated task.

2. FERRITE DIRECTING SYSTEM

The system discussed in the research is a cylindrical ferrite line of a surface wave without a spiral. The transversal section of this system is shown on Fig.1.

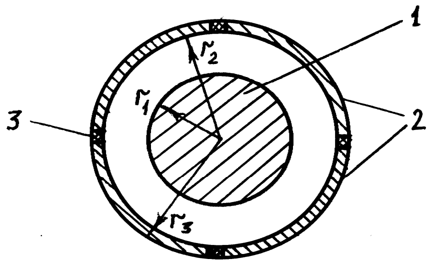

Figure 1. *The transversal section of the ferrite directing system*

It consists of the ferrite bar 1 with a radius r_1, placed in a metallic tube with the longitudinal cuts. The ferrite has dielectric permeability ε_1 and magnetic μ_1. A dielectric 3 with dielectric permeability ε_2 is placed between the metallic strips 2.

The presence of the capacity between the metallic strips formed by longitudinal cuts allows exciting a slow surface wave in a line by the coil of connection. A bar is fastened to the tube by the dielectric pucks.

The transversal electric wave can exist in such a system [1].

For the symmetric transversal electric wave of TE metallic tube with cuts it is possible to present as a tube from an artificial anisotropic dielectric, in which the field constituent E_φ creates the current of displacement on a unit of the length of line

$$I_\varphi = i\omega C_\Pi E_\varphi 2\pi r_2,$$

where C_Π is a linear capacity which consists of the consistently united capacities between the edges of metallic strips,

ω – is circular frequency,

E_φ – transversal constituent of the electric field,

r_2 – internal radius of metallic tube.

In this case the line of the surface wave is presented as a line with equivalent linear parameters L_Π, C_Π. Let's find these linear parameters.

The task should be solved at the following assumptions:

1. A line is a homogeneous segment of waveguide with the surface symmetric wave of TE, having the field constituents in the cylindrical system of co-ordinates H_z, E_φ and H_r (the axis of Z coincides with the axis of line);
2. Fading of wave is small;
3. Thickness of the metallic tube with cuts $(r_3 - r_2) << \lambda_z$, therefore it can be presented by a current's film of displacement;
4. The constituents of fields are changing in time due to a harmonic law.

In the case of deceleration of the wave $K = \sqrt{\omega^2 \varepsilon_0 \mu_0 - \beta^2} = i\sqrt{\beta^2 - \omega^2 \varepsilon_0 \mu_0} = ik$, because phase coefficient $\beta > \omega\sqrt{\varepsilon_0 \mu_0}$. Here ε_0 and μ_0 – accordingly a dielectric and a magnetic permeability of free space.

Consequently, complex amplitudes of the constituents of the field of wave, hurrying in the direction of axis of Z, are described by the following expressions.

In an area $0 \le r \le r_1$

$$H_z = a_1 I_0(k_1 r) e^{-i\beta z},$$
$$H_r = -\frac{1}{k_1^2}\left[-i\beta \frac{\partial H_z}{\partial r}\right] = i a_1 \frac{\beta}{k_1} I_1(k_1 r) e^{-i\beta z},$$
$$E_\varphi = -\frac{1}{k_1^2}\left[-i\omega\mu_1 \frac{\partial H_z}{\partial r}\right] = -i\frac{\omega\mu_1}{k_1} a_1 I_1(k_1 r) e^{-i\beta z}.$$

In an area $r_1 \le r \le r_2$

$$H_z = [a_2 I_0(k_2 r) + b_2 K_0(k_2 r)] e^{-i\beta z},$$
$$H_r = -i\frac{\beta}{k_2} a_2 K_1(k_2 r) e^{-i\beta z},$$
$$E_\varphi = -i\frac{\omega\mu_0}{k_2}[a_2 I_1(k_2 r) - b_2 K_1(k_2 r)] e^{-i\beta z}.$$

In an area $r \ge r_3$

$$H_z = b_3 K_0(k_3 r) e^{-i\beta z},$$
$$H_r = -i\frac{\beta}{k_3} b_3 K_1(k_3 r) e^{-i\beta z},$$
$$E_\varphi = i\frac{\omega\mu_0}{k_3} b_3 K_1(k_3 r) e^{-i\beta z}.$$

In these expressions $I_0(kr)$ and $I_1(kr)$ are the modified functions of Bessel, and $K_0(kr)$ and $K_1(kr)$ are the functions of Macdonald; k_1 – the phase coefficient in an area, occupied by the ferrite; k_2 – the phase coefficient in an area between the ferrite and the metallic tube; k_3 – the phase coefficient in an area out of the line of the surface wave.

At finding of linear capacity $C_П$ let's use the following boundary conditions

$$\left.\begin{aligned} H_{1z} - H_{2z} &= 0 \\ E_{1\varphi} - E_{2\varphi} &= 0 \end{aligned}\right\} \quad \text{at} \quad r = r_1,$$

$$\left.\begin{aligned} H_{2z} - H_{3z} &= I_\varphi \\ E_{2\varphi} - E_{3\varphi} &= 0 \end{aligned}\right\} \quad \text{at} \quad r = r_2 \approx r_3.$$

Using these boundary conditions, we will get the following system of equations for determination of the coefficients a_i, b_i:

$$\left.\begin{aligned} & a_1 I_0(k_1 r_1) = a_2 I_0(k_2 r_1) + b_2 K_0(k_2 r_1), \\ & a_1 \frac{\mu_1}{k_1} I_1(k_1 r_1) = \frac{\mu_0}{k_2}\left[a_2 I_1(k_2 r_1) - b_2 K_1(k_2 r_1)\right], \\ & a_2 I_0(k_2 r_2) + b_2 K_0(k_2 r_2) - b_3 K_0(k_2 r_2) = \\ & \qquad = \frac{\omega^2}{k_2}\mu_0 2\pi r_2 \left[a_2 I_1(k_2 r_2) - b_2 K_1(k_2 r_2)\right] C_П, \\ & - a_2 I_1(k_2 r_2) + b_2 K_1(k_2 r_2) = b_3 K_1(k_2 r_2). \end{aligned}\right\} \tag{1}$$

The given system of equations is brought by the method of Gauss to the three-cornered form. The last equation of the new system looks like as follows:

$$b_3 \left\{ \frac{K_0(k_2 r_2)}{M}\left[K_1(k_2 r_2) + \frac{B}{A} I_1(k_2 r_2)\right] - K_1(k_2 r_2) \right\} = 0, \tag{2}$$

where

$$\begin{aligned} M = & \left[K_0(k_2 r_2) + \frac{\omega^2}{k_2}\mu_0 2\pi r_2 C_П K_1(k_2 r_2)\right] - \\ & - \frac{B}{A}\left[I_0(k_2 r_2) + \frac{\omega^2}{k_2}\mu_0 2\pi r_2 C_П I_1(k_2 r_2)\right], \end{aligned} \tag{3}$$

$$\frac{B}{A} = \frac{K_1(k_2 r_1)}{I_1(k_2 r_1)} = \frac{\dfrac{\mu_1}{\mu_0}\dfrac{k_2}{k_1}\dfrac{K_0(k_2 r_1)}{K_1(k_2 r_1)}\dfrac{I_1(k_1 r_1)}{I_0(k_1 r_1)} + 1}{\dfrac{\mu_1}{\mu_0}\dfrac{k_2}{k_1}\dfrac{I_0(k_2 r_1)}{I_1(k_2 r_1)}\dfrac{I_1(k_1 r_1)}{I_0(k_1 r_1)} - 1}. \tag{4}$$

Coefficient $b_3 \neq 0$; otherwise the field outside and it means that inside a line as well, is absent. Consequently, expression in the figured brackets of equation (2) is equal to the zero

$$\frac{K_0(k_2 r_2)}{M}\left[K_1(k_2 r_2) + \frac{B}{A} I_1(k_2 r_2)\right] - K_1(k_2 r_2) = 0. \tag{5}$$

Using the expressions (3) and (5), we find a linear capacity:

$$C_{\Pi} = \frac{1}{2\pi r_2^2 \mu_0 \omega^2 K_1(k_2 r_2) I_1(k_2 r_2)\left[\frac{A}{B}\frac{K_1(k_2 r_2)}{I_1(k_2 r_2)} + 1\right]}. \qquad (6)$$

A relation A/B is determined from expression (4) by transposition of the places of numerator and denominator.

Usually diameter of aerial is lesser than wavelength and $kr \ll 1$ for all the areas of aerial; therefore the equation (6) can be simplified therefore, using the known expressions:

$$\left.\begin{aligned} &I_0(x) \approx 1, \\ &I_1(x) \approx x/2, \\ &K_1(x) \approx 1/x, \\ &K_0(x) \approx \ln(1{,}12/x). \end{aligned}\right\} \quad \text{at} \quad x \ll 1$$

So,

$$C_{\Pi} \approx \frac{1}{\pi r_2^2 \mu_0 \omega^2} \frac{\frac{\mu_1}{\mu_0}\frac{(k_2 r_1)^2}{2}\ln\frac{1{,}12}{k_2 r_1} + 1}{\frac{r_1^2}{r_2^2}\left(\frac{\mu_1}{\mu_0} - 1\right) + \frac{\mu_1}{\mu_0}\frac{(k_2 r_1)^2}{2}\ln\frac{1{,}12}{k_2 r_1} + 1}. \qquad (7)$$

From the got expression it is possible to get a critical frequency, putting $k_2 = 0$,

$$\omega_{\kappa p}^2 = \frac{1}{C_{\Pi} \pi r_2^2 \mu_0 \left\{\frac{r_1^2}{r_2^2}\left[\frac{\mu_1}{\mu_0} - 1\right] + 1\right\}}. \qquad (8)$$

On frequencies below the critical the surface wave in a line can’t be excited.

From the formula (8) it is evidently, that on a critical frequency resonance takes place in an equivalent contour, formed by the capacity $C = C_{\Pi} \cdot \ell$ and inductance

$$L = \pi r_2^2 \mu_0 \left\{\frac{r_1^2}{r_2^2}\left[\frac{\mu_1}{\mu_0} - 1\right] + 1\right\}\frac{1}{\ell}, \qquad (9)$$

where ℓ is the length of line.

The sought for expression to the linear inductance of L_{Π} can be got by the division of expression (9) on the length of line ℓ:

$$L_{\Pi} = \pi r_2^2 \mu_0 \left\{\frac{r_1^2}{r_2^2}\left[\frac{\mu_1}{\mu_0} - 1\right] + 1\right\}\frac{1}{\ell^2}. \qquad (10)$$

The expressions (7) and (10) that we have got allow us to make the electric calculation of this directing system.

References

[1] Shelkovnikov Vladimir. Moving Objects Information Provision Quality Growth Due to the Electrodynamics’ Screens Application in Ferrite Receiving Antennae, *Transport and Telecommunication*, 2002, Vol. 3, No. 1, pp. 104-113.

MODELLING AND CHARACTERISTICS OF SELF-SIMILAR SYSTEMS WITH HIGH QUALITY FILTRATION

Vitaly Yeremeyev, Sergey Sharkovsky

In this paper a new conception of digital self-similar systems design, which time and frequency characteristics are satisfied to the specification, but its realization is simpler comparing with standard procedures, is offered. The basic cascade connection of the unified biquades is chosen. There are added two matched biquades. Usually no more than ten constants should be stored in memory. The program realization as well as hardware realization of selfsimilar systems is simpler than traditional. Biquades coefficients are found by local optimization.

The DLPF design example of the 10-th order, which characteristics are compared with the characteristics of Chebyshev's filters with the same order and same requirements, is shown in the article; DLPF of the 14-th order with specified requirements of step response is shown as well. The characteristics and parameters of the synthesized filters are investigated.

Keywords: self-similar systems, digital filters, time and frequency characteristics requirements, transfer function of multiple poles, parameter of time and frequency characteristics optimization, biquad realization

МОДЕЛИРОВАНИЕ И ХАРАКТЕРИСТИКИ САМОПОДОБНЫХ СИСТЕМ ВЫСОКОКАЧЕСТВЕННОЙ ФИЛЬТРАЦИИ

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Предложена новая концепция синтеза цифровых структурно самоподобных систем, частотные и временные характеристики которых удовлетворяют заданным, а реализация более простая по сравнению со стандартными процедурами. Базовым выбрано каскадное соединение одинаковых биквадов с добавлением двух согласующих. Для программной реализации хранится не более десяти констант. И программная и аппаратная реализация существенно проще традиционных. Коэффициенты биквадов найдены локальной оптимизацией.

Приведен пример синтеза ЦФНЧ 10 порядка, характеристики которого сравниваются с характеристиками фильтра Чебышева того же порядка и при тех же требованиях. Показан синтез ЦФНЧ 14 порядка с заданными требованиями к переходной характеристике. Приведены характеристики и параметры синтезированных фильтров.

Ключевые слова: структурно самоподобные системы, цифровые фильтры, требования к частотным и временным характеристикам, кратные полюса передаточной функции, оптимизация параметров частотных и временных характеристик, биквадная реализация

1. ВВЕДЕНИЕ

Проектирование цифровых рекурсивных фильтров, которые удовлетворяют заданным требованиям к нескольким характеристикам и одновременно с этим просты в реализации по сравнению с известными, всегда актуально для специалистов по цифровой обработке сигналов. При проектировании цифровых рекурсивных фильтров традиционно используют аппроксимации Баттерворта, Чебышева, Бесселя, изоэкстремальные и многие другие. Популярная их реализация – каскадная, т.е. биквадная (звенья второго порядка), реализация, которая в общем случае выглядит следующим образом (см. рис. 1).

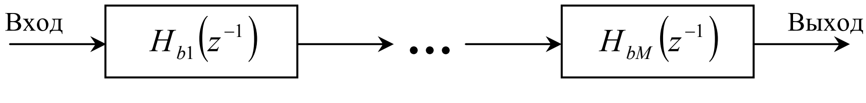

***Рис. 1**. Биквадная реализация*

Здесь для фильтров четных порядков $M = N/2$ и $H_{bk}\left(z^{-1}\right) = \dfrac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$, при $k = 1...M$. Для фильтров нечетных порядков $M = (N+1)/2$, $H_{bk}\left(z^{-1}\right) = \dfrac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$, при $k = 1...(M-1)$ и $H_{bM}\left(z^{-1}\right) = \dfrac{b_{M0} + b_{M1}z^{-1}}{1 + a_{M1}z^{-1}}$. Очевидно, нам потребуется хранить одновременно $5M$ значений коэффициентов для фильтров четных порядков и $5M-2$ коэффициентов для фильтров нечетных порядков, что при больших N может оказаться неприемлемым. В ряде случаев для аппаратной или программной реализации очень хотелось бы иметь унифицированные субзвенья в составе такого каскадного соединения.

В данной статье рассматривается новая концепция проектирования цифровых рекурсивных фильтров с помощью биквадной реализации, которая в общем случае выглядит так (рис. 2):

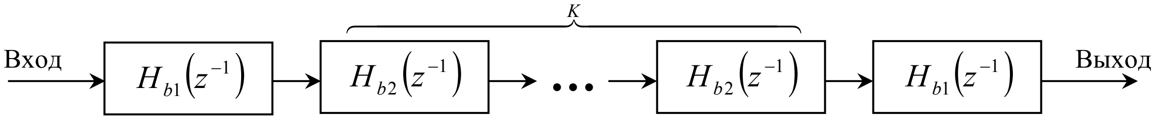

Рис. 2. *Реализация с использованием унифицированных звеньев*

В этом случае $H_{b1}\left(z^{-1}\right) = \dfrac{b_{10} + b_{11}z^{-1} + b_{12}z^{-2}}{1 + a_{11}z^{-1} + a_{12}z^{-2}}$, $H_{b2}\left(z^{-1}\right) = \dfrac{b_{20} + b_{21}z^{-1} + b_{22}z^{-2}}{1 + a_{21}z^{-1} + a_{22}z^{-2}}$ и $K > 0$. Как видно из рис. 2, общее количество множителей, сохраняемых в ПЗУ, всегда не более десяти при любом порядке фильтра, а аппаратная или программная реализация значительно проще. Как будет показано ниже, при использовании такой системы можно получать хорошие частотные и временные характеристики фильтра.

2. СИНТЕЗ ЦФНЧ С ЗАДАННЫМИ ТРЕБОВАНИЯМИ К АЧХ

Покажем, что с помощью нового метода можно получать амплитудно-частотные характеристики (АЧХ) не хуже чем у фильтров Чебышева первого рода того же порядка.

Для нахождения коэффициентов биквадов $H_{b1}\left(z^{-1}\right)$ и $H_{b2}\left(z^{-1}\right)$ был выбран итерационный метод локальной оптимизации с использованием стандартных функций пакета MATLAB [3], который уже использовался авторами статьи ранее [1]. Входными данными блока оптимизации могут быть требуемые параметры как амплитудно-частотной характеристики, так и одной из временных характеристик (в нашем случае – переходной).

В качестве примера сравним АЧХ фильтра Чебышева первого рода 10 порядка с АЧХ предлагаемого фильтра (рис. 2). Ясно, что $K = 3$. Коэффициенты биквадов $H_{b1}\left(z^{-1}\right)$ и $H_{b2}\left(z^{-1}\right)$, полученные с помощью указанной оптимизации, равны:

```
b1 =   0.29192246006399   0.15211250314525   0.13077791987071
b2 =   0.67759117822912  -0.54642925473671   0.67634985544519
a1 =   1.00000000000000  -0.72246900489605   0.24168173957638
a2 =   1.00000000000000  -0.93958856592357   0.80499694921226
```

При проектировании были выбраны следующие параметры АЧХ: нормированная частота среза $\omega_1 = 0.3$, затухание на контрольной частоте $\omega_k = 0.35$ $a_{\min} \geq 40dB$. Для фильтра Чебышева было выбрано $\omega_1 = 0.3$, $a_{\max} = 0.254$. Сопоставляемые АЧХ показаны на рис. 3.

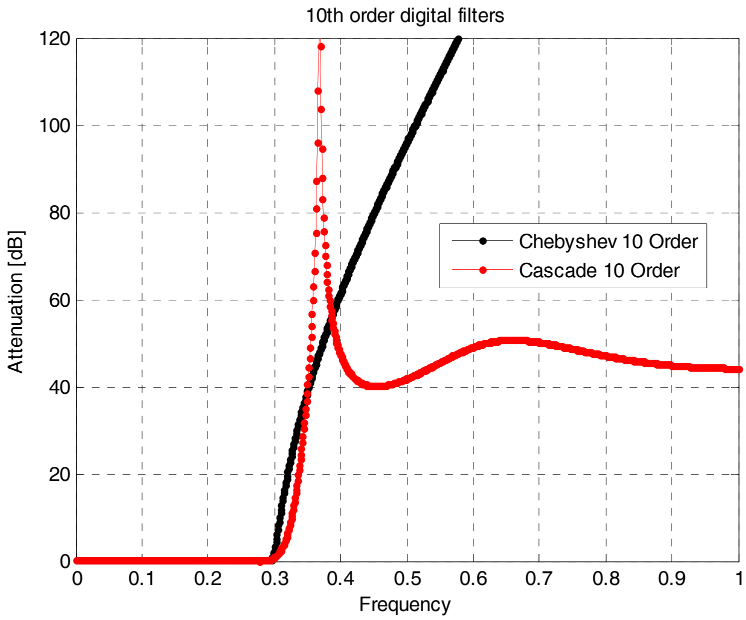

Рис. 3. *Характеристики затухания сопоставляемых фильтров 10 порядка*

Как видно из графика рис. 3, АЧХ предлагаемого фильтра не уступает в избирательности фильтру Чебышева первого рода того же порядка. Однако при синтезе должна быть решена проблема пульсаций АЧХ в полосе пропускания, так как при каскадном включении биквадов их затухания суммируются. В нашем случае почти все биквады одинаковые, поэтому мы будем суммировать одни и те же кривые затухания. Кроме того, понятно, что в полосе пропускания будет отсутствовать характерная для фильтров Чебышева равноволновость пульсаций АЧХ (рис. 4). Эта проблема решается включением дополнительных биквадных структур с характеристикой затухания по форме инверсной к основным. На рис. 2 такие структуры показаны крайними.

Интересно сравнить и временные характеристики. На рис. 5 показаны переходные характеристики сопоставляемых фильтров. Видно, что первый выброс у фильтров Чебышева и время нарастания у них несколько выше. Это объясняется перераспределением энергии спектральных составляющих, сосредоточенных в полосе задерживания. Таким образом, переходная характеристика у синтезированного фильтра (рис. 2) лучше, чем у фильтра Чебышева того же порядка.

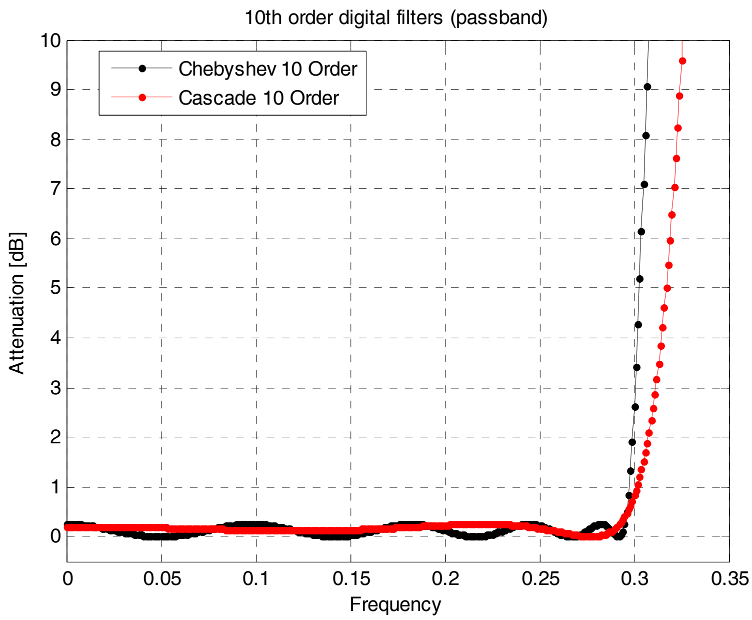

Рис. 4. *Характеристики затухания сопоставляемых фильтров 10 порядка в полосе пропускания*

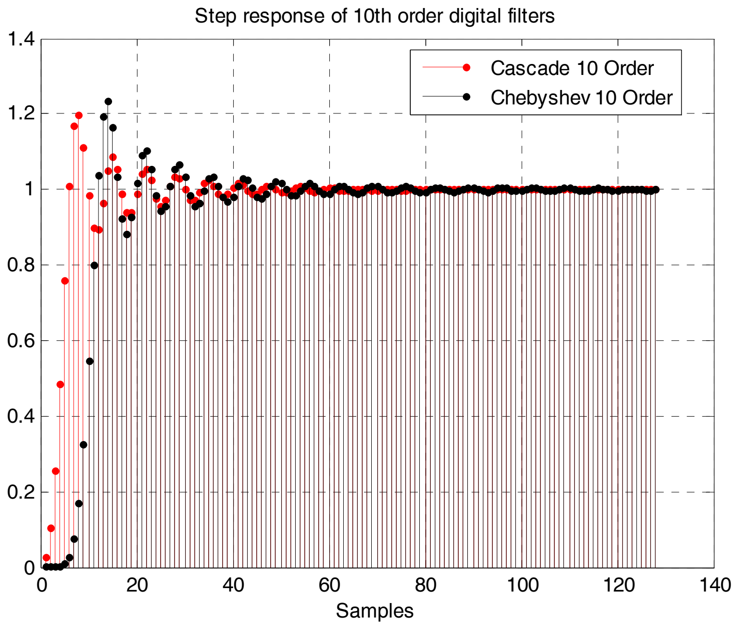

Рис. 5. *Переходные характеристики сопоставляемых фильтров 10 порядка*

3. СИНТЕЗ ЦФНЧ С ЗАДАННЫМИ ТРЕБОВАНИЯМИ К ПЕРЕХОДНОЙ ХАРАКТЕРИСТИКЕ

В [1] авторами был показан синтез ЦФ с передаточными функциями, имеющими кратные корни в их знаменателе. Доказано, что наличие кратных корней в знаменателе гарантирует переходные характеристики с экстремально низкими выбросами (до 0.5%). Система на рис. 2 как раз и имеет кратные корни, поэтому логично предположить, что у нее также можно получить переходные характеристики с исключительно низкими выбросами. Задав при оптимизационном поиске коэффициентов соответствующие требования к переходной характеристике, в результате ряда итераций получен фильтр 14 порядка с характеристикой затухания, показанной на рис. 6.

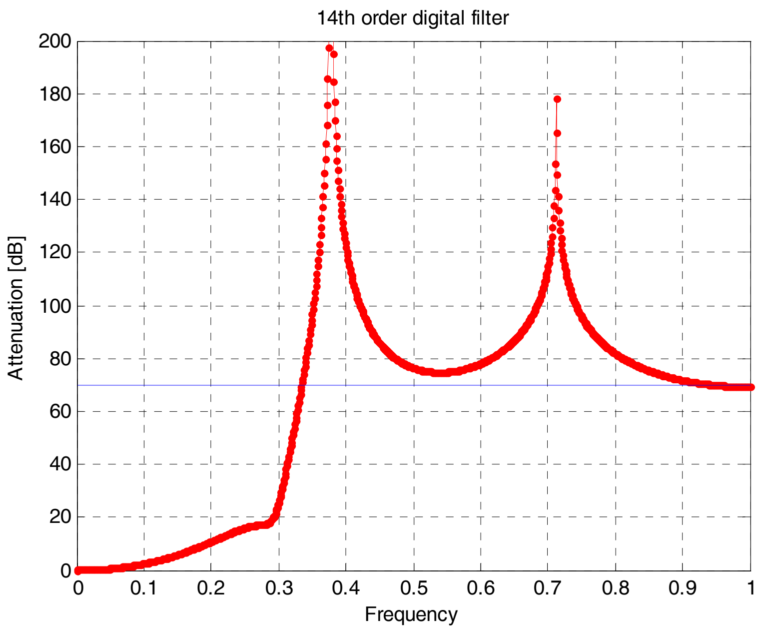

Рис. 6. *Характеристики затухания синтезированного фильтра 14 порядка*

На рисунке видно, что кривая в полосе пропускания имеет нехарактерную для традиционной фильтрации форму [1]. Затухание на контрольной частоте равно $a_{\min} = 70dB$. Переходная характеристика приведена на рис. 7 и 8. Уровень пульсаций ее не превышает одного процента.

Коэффициенты, полученные в ходе синтеза, следующие:

```
b1 =   0.07797565134107   0.09647941636901   0.07797565134107
b2 =  -0.53707717629099   0.40385739734949  -0.53707717629099
a1 =   1.00000000000000  -1.13794825990780   0.89505573191871
a2 =   1.00000000000000  -0.65138727471858   0.17295287696412
```

Стоит отметить, что в данном случае достаточно хранить в ПЗУ значения восьми коэффициентов умножителей, т.к. $b_{10} = b_{12}$ и $b_{20} = b_{22}$.

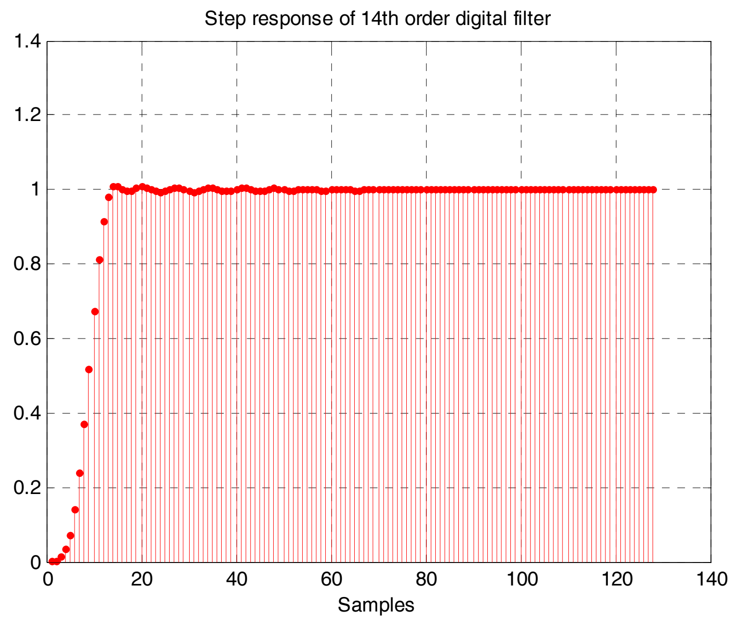

Рис. 7. *Переходная характеристика синтезированного фильтра 14 порядка*

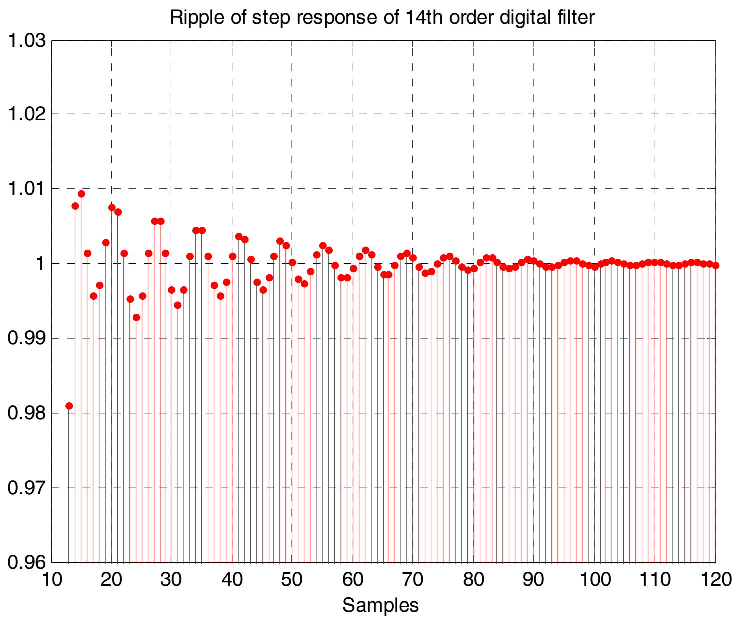

Рис. 8. *Детализированная переходная характеристика синтезированного фильтра 14 порядка*

ЗАКЛЮЧЕНИЕ

Показана новая концепция синтеза ЦФ (рис. 2). Используя данную систему, можно получать как высокоизбирательные фильтры, которые не уступают известным, а иногда значительно их превосходят (рис. 3–5). Эти фильтры обеспечивают минимальные или заданные пульсации переходных характеристик (рис. 6–8).

Главное достоинство – это простота реализации и малое количество используемой рабочей памяти (ПЗУ), так как число операндов, которые следует хранить, не более десяти.

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RESEARCH OF CHARACTERISTICS OF ROBUST TUNEABLE SUPER NARROW-BAND FILTRATION SYSTEMS

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The article describes the method of pass band and stop band structures synthesis, which doesn't use the traditional band frequency transformation.

Key words: characteristics, modelling, research, reliability, computational effective structures.

1. INTRODUCTION

During synthesis of tuneable pass band or stop band digital filters the complicated enough bilinear frequency transformations are almost always used, that often leads to doubling of the prototype order. Moreover, at attempt to realize the super narrow-band pass- or stop band filters the double factorization of numerator's and denominator's polynomials of prototype transfer function is required - during conversion from the prototype to digital structure and during decomposition of last one to bilines [1]. It negatively affects the accuracy of transfer function's poles' and zeros' calculations and, finally, influences on inexact reproduction of filter's frequency characteristics.

2. DESCRIPTION OF RESEARCH

The article describes the method of pass band and stop band structures synthesis, which doesn't use the traditional band frequency transformation.

The usual band frequency transformation is described by the expression (1):

$$\Omega = k\frac{\alpha - \cos\left(\pi \cdot \widetilde{\Omega}\right)}{\sin\left(\pi \cdot \widetilde{\Omega}\right)}, \qquad (1)$$

$$k = ctg\left(\frac{\pi}{2}\left(\widetilde{\Omega}_1 - \widetilde{\Omega}_{-1}\right)\right), \qquad (2)$$

$$\alpha = \cos\left(\pi \cdot \widetilde{\Omega}_0\right),$$

$$\widetilde{\Omega}_i = 2\frac{f_i}{f_s}, \qquad (3)$$

where k sets the band width and a – position of the central frequency.

The known transformation of transfer function of low-frequency prototype into a digital one is described by the expression (2):

$$H(p) \Rightarrow H\left(k\frac{1-2z^{-1}+z^{-2}}{1-z^{-2}}\right) \qquad (4)$$

Let's consider the following approach. During shifting the zero frequency of the filter to the central frequency Ω_0 of the projected filter $S(\Omega) \Rightarrow S(\Omega - \Omega_0)$ the following transformation of its delay element is occurred:

$$\left[z^{-1} = e^{-j\pi\cdot\tilde{\Omega}}\right] \Rightarrow \left[e^{-j\pi\cdot(\tilde{\Omega}-\tilde{\Omega}_0)} = z^{-1}\cdot e^{j\pi\cdot\tilde{\Omega}_0}\right] \qquad (5)$$

Thus, it is necessary to include the complex multiplier of $A = e^{j\pi\cdot\tilde{\Omega}_0}$ sequentially to the each delay element in the structure of low-frequency digital filter. In this case, it controls or changes the position of central frequency of the band filter during tuning. The width of a pass band of the last one, as well as in case of LF-filter, is defined by *k*-parameter.

As the article considers the opportunity of synthesis of super narrow-band digital filters, it is reasonable to use special biline structures [2]. The transformation of biline units of a general form is shown on fig. 1.

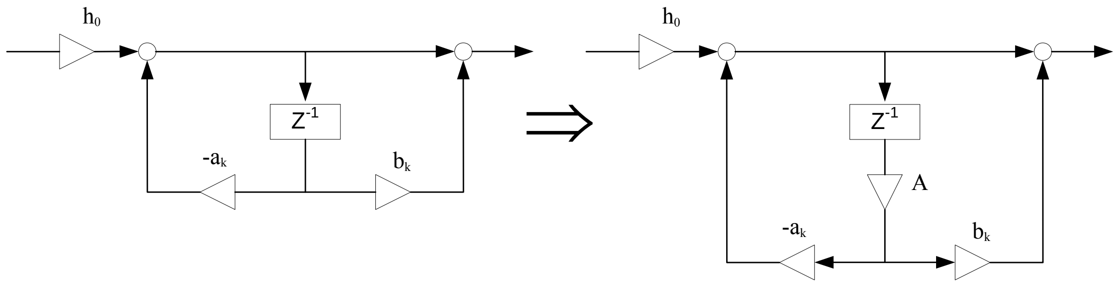

Fig. 1. *Transformation from LFF to band digital filter*

The obvious advantage of the new tuneable filters in comparison to the traditional realizations consists in practically small modification of the structure of the low-frequency digital prototype.

Let's consider a variant of realization of the band tuneable filter. As $A = e^{j\pi\cdot\tilde{\Omega}_0}$, it is obvious that the band filter with central normalized frequency equal to 0.5 is generated simply enough: in this case, that leads to rearrangement of real and imaginary outputs of a delay element.

3. EXAMPLES

Let's show the characteristics of band tuneable biline filters with very narrow band on the example of elliptical filter [2]. Set the next requirements to the frequency response:

1. Filter order N = 15;
2. Pass band ripple a_{max} = 0.05 dB.
3. Normalized pass band width $\Delta\tilde{\Omega} = 2\cdot 10^{-11}$;
4. Normalized central frequency $\tilde{\Omega}_0 = 4\cdot 10^{-11}$.

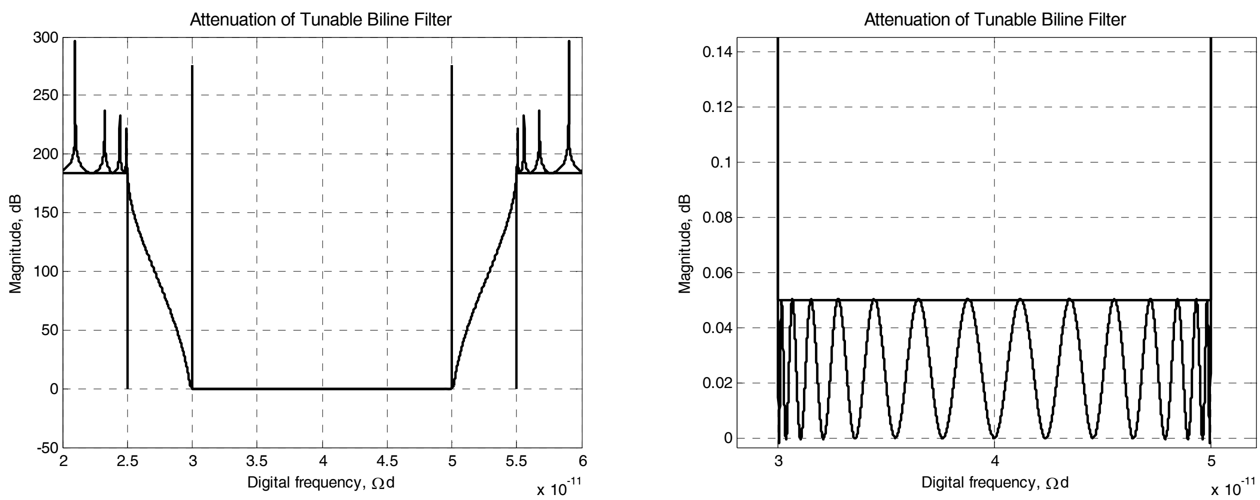

Fig. 2. *Frequency characteristics of tuneable pass band filter*

1. Filter order N = 41;
2. Pass band ripple a_{max} = 0.05 dB.
3. Normalized pass band width $\Delta\widetilde{\Omega} = 2 \cdot 10^{-12}$.
4. Normalized central frequency $\widetilde{\Omega}_0 = 4 \cdot 10^{-12}$.

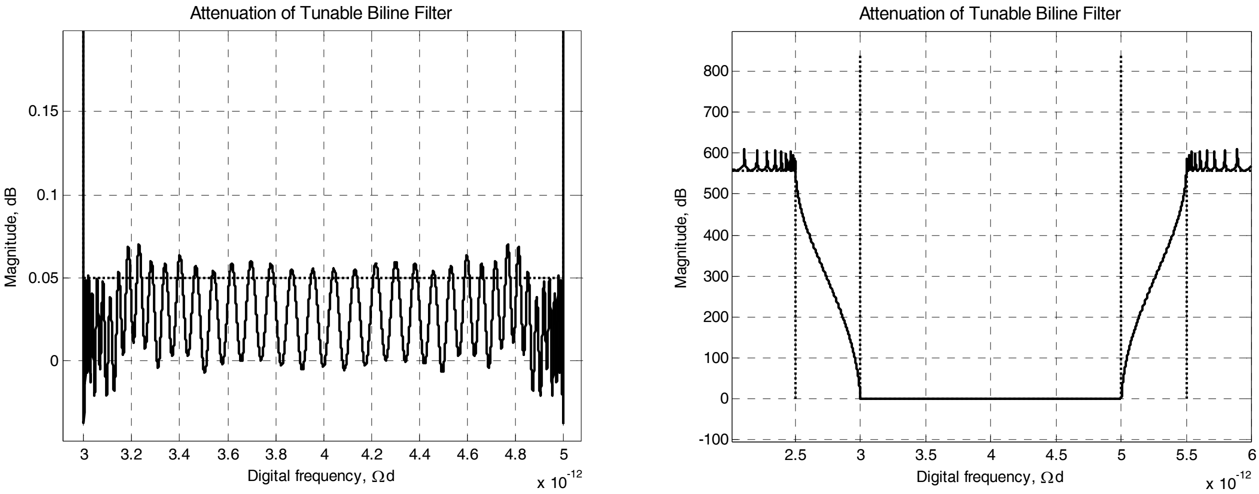

Fig. 3. *Frequency characteristics of tuneable pass band filter in the area of critical frequencies.*

CONCLUSIONS

The method of synthesis of super-narrow tuneable pass band and stop band digital filters is offered. The frequency characteristics of these structures practically don't differ from characteristics of traditional complete biline structures in the narrow frequency bands.

The basic opportunity of new digital filters synthesis, which central frequency is controlled by the multiplier, identical in any of biline, is shown in the investigation.

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THE ANALYSIS, SYNTHESIS AND MODELLING OF ROBUST NARROW-BAND FILTRATION SYSTEMS

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The article describes the special method of synthesis of polynomial supernarrow-band structures, using traditional frequency transformation.

Keywords: characteristics, modelling, research, sensitivity, robustness, effective structures

1. INTRODUCTION

During synthesis of computational-effective structures [1] the special Z-transformation is used, as a result the digital filters with only recursive parts are synthesized. Poles of such calculated transfer function of the digital filter basically should be different from the poles calculated by a usual method.

2. RESEARCH DESCRIPTION

The analogue prototypes, described by a transfer function as a unit, divided on a polynomial, have corresponding bilinear transfer functions in digital area, numerators of which distinct from unit at bilinear transformation. For example, the Butterworth and Chebyshev I-type prototypes correspond to these prototypes. In general, the biline realizations [2] of such super narrow-band structures are described by the following transfer function (1):

$$H(z^{-1}) = H_0 \frac{1+z^{-1}}{1+a_0 z^{-1}} \prod_{k=1}^{N-1/2} \frac{b_{0k} + b_{1k} z^{-1}}{1+a_{1k} z^{-1}} . \tag{1}$$

For polynomial prototypes the numerators of bilines transfer functions are relatively simple (2):

$$H(z^{-1}) = H_0 \frac{1+z^{-1}}{1+a_0 z^{-1}} \prod_{k=1}^{N-1/2} \frac{1+z^{-1}}{1+a_{1k} z^{-1}} . \tag{2}$$

The class of the digital filters described by transfer function (2) provides demanded normalized bands of the order of $1 \cdot 10^{-14}$-$1 \cdot 10^{-15}$. The poles of biline transfer functions, as parts of expression (2), form the complex conjugate pairs.

For super narrow-band filters, it is desirable to use the simplified biline structures, numerators of which transfer function are units. In this case the modified transfer function is described by the expression (3):

$$H(z^{-1}) = H_0 \frac{1+z^{-1}}{1+a_0 z^{-1}} \prod_{k=1}^{N-1/2} \frac{1}{1+a_{1k} z^{-1}} . \tag{3}$$

Such approach leads to the following questions:

1. What is the qualitative difference between frequency characteristics of the modified structures and the usual ones in a whole working frequencies range?
2. How much the modification simplifies the realization?

Let's estimate the frequency characteristics of new filters. Set the next requirements to the filter:

Filter's order N = 15; Normalized pass band width $\tilde{\Omega}_1 = 0.5$.

Pass band ripple $a_{max} = 0.1$ dB.

As shown on Fig. 1, the frequency characteristics of the new filter with requirements above is too far from ideal and doesn't satisfy them.

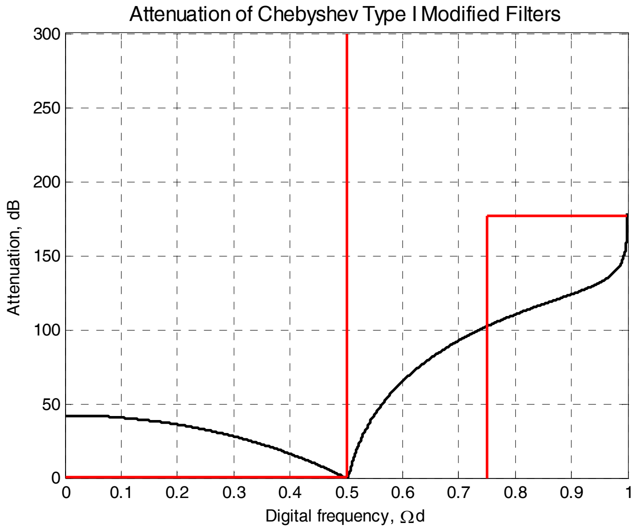

Fig. 1. *Frequency characteristics of the filter with the simplified numerator of transfer function*

However, at narrow normalized bands of the order of $\tilde{\Omega}_1 = 1 \cdot 10^{-3}$ and less the characteristics of modified filter in the pass band coincide with the biline filter's characteristics (Fig. 2).

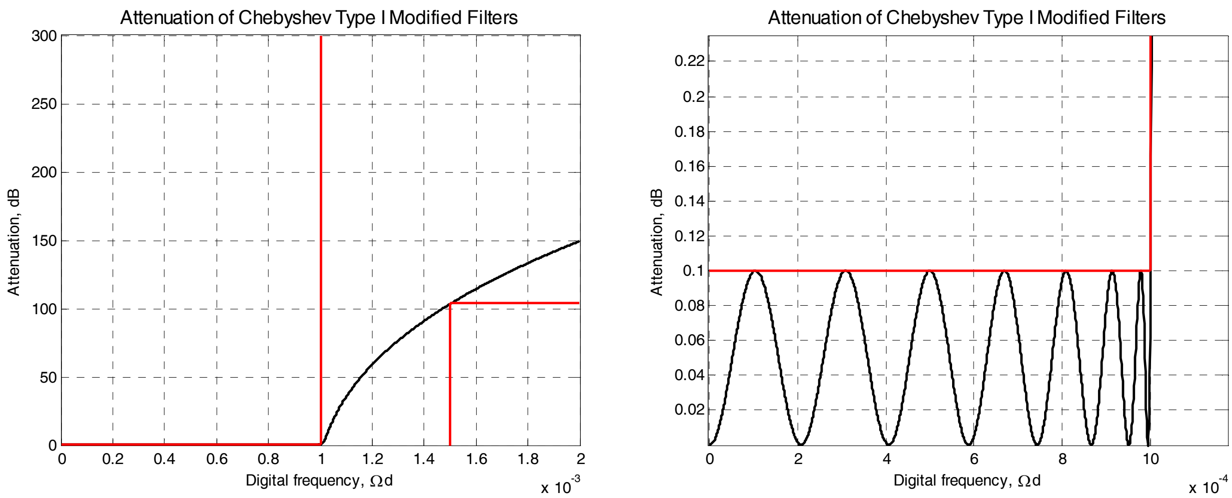

Fig. 2. *Frequency characteristics of modified biline filter with normalized pass band width of* $\tilde{\Omega}_1 = 1 \cdot 10^{-3}$

Let's estimate the minimum possible realizable pass band width of polynomial digital structures. Reduce the pass band until $\tilde{\Omega}_1 = 2 \cdot 10^{-12}$. Then, the beginning of distortion of frequency characteristics is shown on Fig. (3).

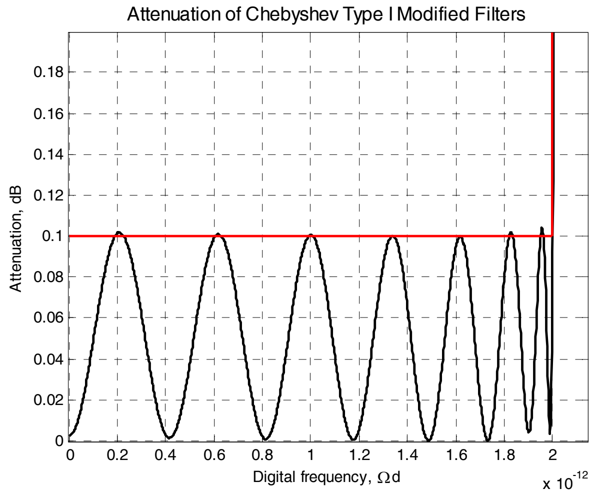

Fig. 3. *Frequency characteristics of modified biline filter with normalized pass band width of* $\widetilde{\Omega}_1 = 2\cdot 10^{-12}$

It has to be remarked that frequency characteristics of standard and computational-effective bilines practically aren't different in the super-narrow frequency range (Fig. 4).

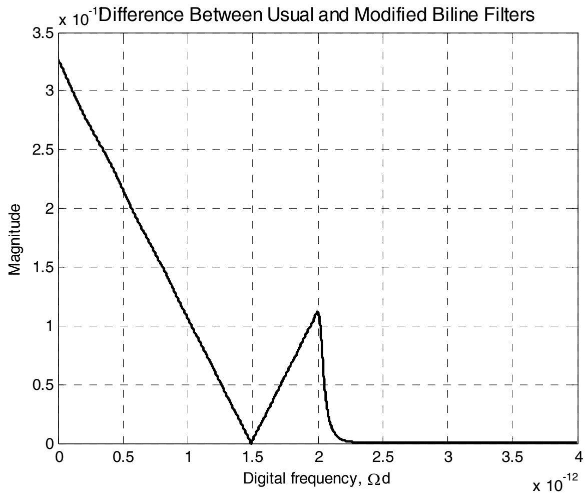

Fig. 4. *Difference between frequency responses of usual and modified biline filters in the pass band*

As it was already mentioned above, the poles of transfer functions of projected bilines form the complex conjugate pairs. It allows conducting the modelling of their work in a real time very effectively.

Let's consider the next bilines pair (Fig. 5).

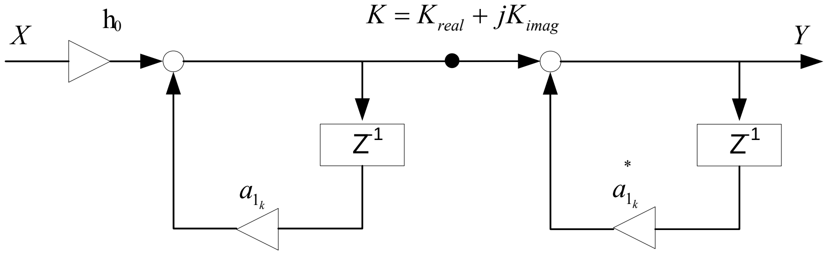

Fig. 5. *Structure scheme of pair of computational effective bilines*

Let's describe the work of each biline in real time, subject to that input X and output Y of pair – is purely real. We shall mean the connection of bilines as K point that has both real and imaginary parts. For the first biline:

$$K_{real} + jK_{imag} = X + (\alpha + j\beta)(M_{1real} + jM_{1imag})$$

$$K_{real} + jK_{imag} = X + \alpha(M_{1real} + M_{1imag}) - M_{1imag}(\alpha + \beta) +$$

$$+ j\big(\alpha(M_{1real} + M_{1imag}) + M_{1real}(\beta - \alpha)\big)$$

Similarly for the second biline:

$$K_{real} + jK_{imag} + (\alpha - j\beta)M_{2real} = Y$$

$$K_{real} + jK_{imag} + \alpha \cdot M_{2real} - j\beta \cdot M_{2real} = Y$$

But so as the exit of the pairs is real, then: $K_{real} + \alpha \cdot M_{2real} = Y$

As a result, the pair of biline units with complex conjugate multipliers is modified into the next structure (Fig. 6):

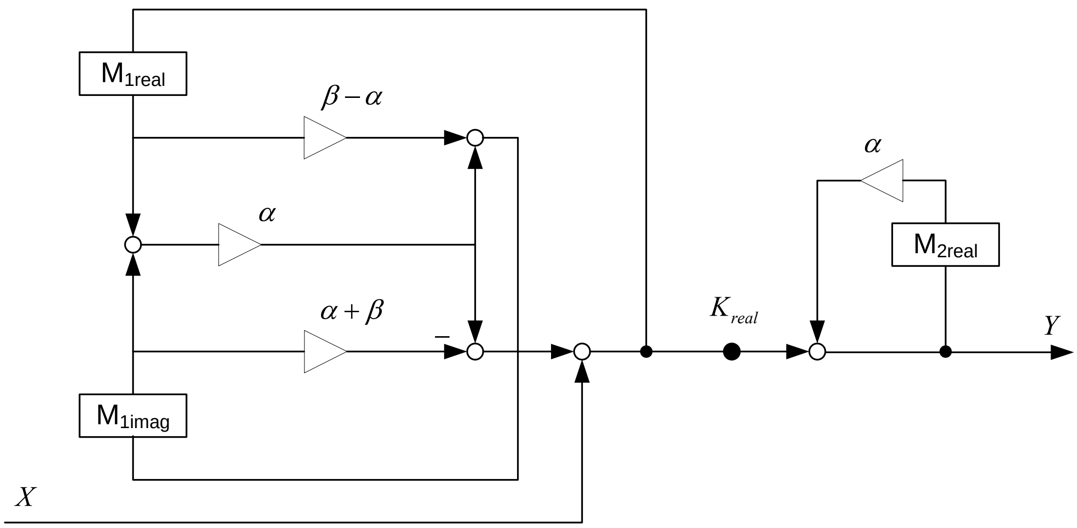

Fig. 6. *Structure scheme of computational-effective bilines pair*

As the figure 6 indicates, the bilines pair of super narrow band digital filter consists of 4 real multipliers, 5 adders and 3 delay elements. That is the one of minimum possible forms of realization, shown on the fig. 5.

As per researches in article [3], let's discuss the possibility of implementation of pass band and stop band super-narrow band digital filters using the approach described above. Indeed, there should be no troubles to realize the tuneable band digital filter, which transfer function has only unit in the numerator.

Thus, correspondingly to [3], the suggested transfer function of tuneable computational-effective band filters should looks like as expression (4):

$$H(z^{-1}) = H_0 \frac{1 + A \cdot z^{-1}}{1 + A \cdot a_0 z^{-1}} \prod_{k=1}^{N-1/2} \frac{1}{1 + A \cdot a_{1k} z^{-1}} \qquad (4)$$

where $A = e^{j\pi \cdot \tilde{\Omega}_0}$ and Ω_0 is the central frequency of the projected filter.

Let's estimate the frequency characteristics of band computational-effective filters. Let's set the next requirements to the filter:

1. Filter order N = 16;
2. Pass band ripple a_{max} = 0.05 dB.
3. Normalized pass band width $\Delta\widetilde{\Omega} = 2 \cdot 10^{-11}$;
4. Normalized central frequency $\widetilde{\Omega}_0 = 4 \cdot 10^{-3}$.

As shown on Fig.7, the frequency characteristics of the band filter with requirements above completely satisfy the requirements even in narrow frequency area.

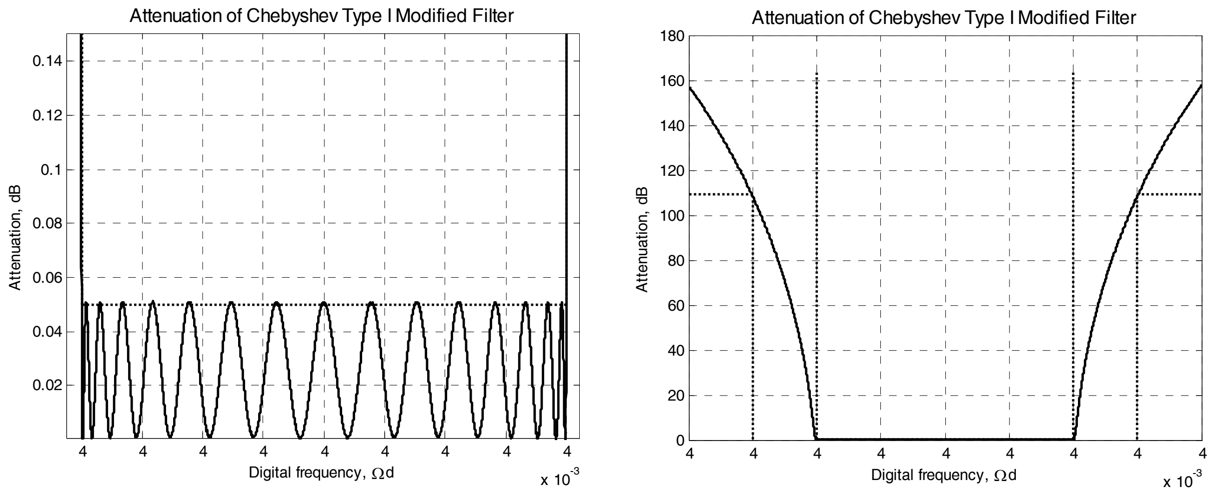

***Fig. 7.** Attenuation of computational-effective modified narrow-band digital filter*

Thus, the realization of band computational-effective digital filters is quite simple. The structure modification is not so obvious, as in the case of LFF or HFF, but anyway some is simpler than in usual traditional case.

CONCLUSIONS

The method of synthesis of computational-effective super narrow-band digital filters without use of special frequency transformations [1] is offered.

Procedure of modelling of a basic element of structure in real time is shown. The frequency characteristics in the field of a pass band practically coincide precisely with calculated in the usual mode.

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